ME 233 – Advanced Control II
Lecture 16
Disturbance Observers

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UC Berkeley

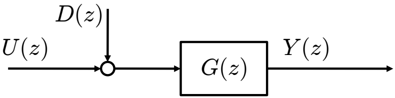
March 29, 2016

Outline

Outline

Motivation

Consider the following plant structure



The signals are:

$$\begin{aligned} U(z) &: \text{ control input} \\ D(z) &: \text{ disturbance} \\ Y(z) &: \text{ output} \end{aligned}$$

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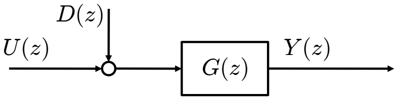

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The goal is to cancel the effect of $D(z)$ on $Y(z)$

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- Let the plant be given by the transfer function $G_n(z)$, which is minimum phase (i.e. its poles and zeros are strictly inside the unit disk in the complex plane)

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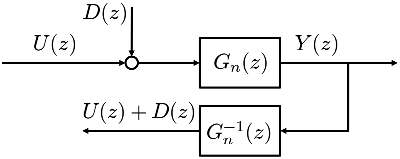
- Use an inverse plant to reconstruct $U(z) + D(z)$:



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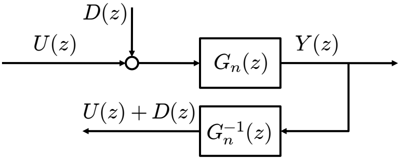
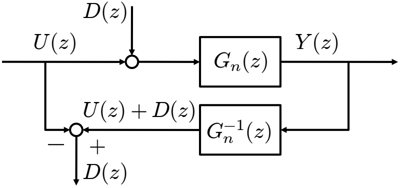


- Subtract $U(z)$ to reconstruct $D(z)$:



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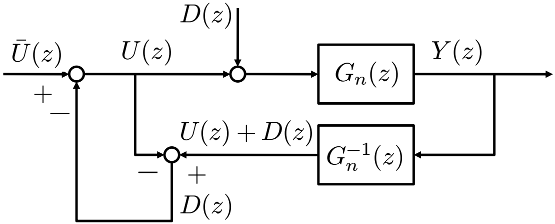
- Ideally, we would subtract the reconstructed value of $D(z)$ from $U(z)$



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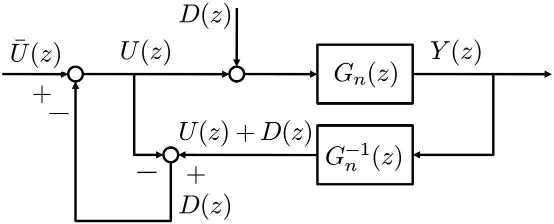

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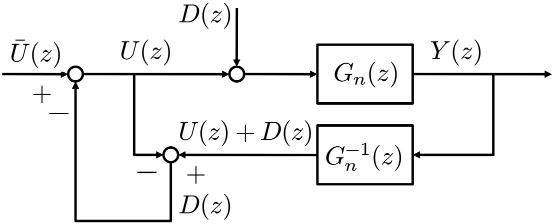

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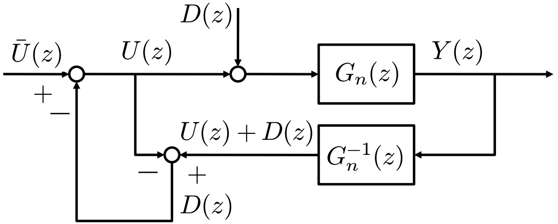

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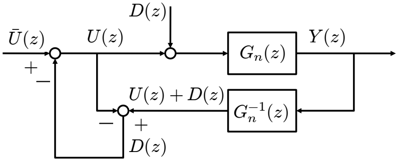
This controller structure would reconstruct $D(z)$ then subtract it from $U(z)$ so that the effect of the disturbance is exactly canceled

$\Rightarrow$ This would be useful as an inner loop of a larger control scheme, BUT...

Motivation—Problems



The control structure has some problems that should be resolved in order for it to be useful:

- Since $G_n^{-1}(z)$ is typically not proper, it is not realizable

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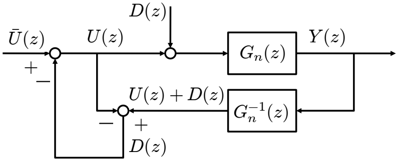

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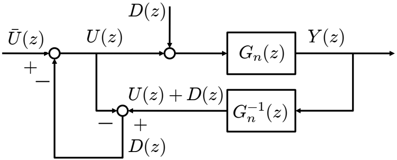

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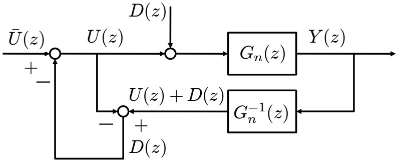

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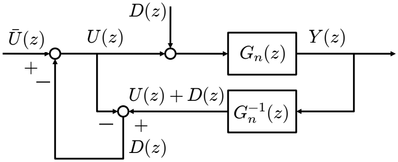

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- The system being controlled might not be exactly as given by the model $G_n(z)$
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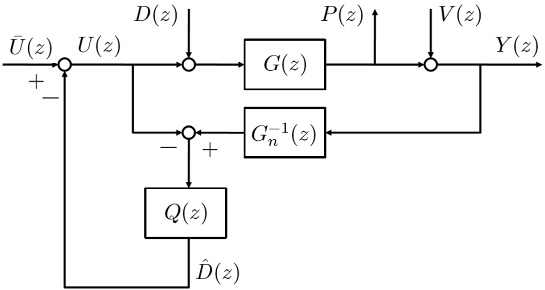
- The block diagram above is not well-posed and, in particular, $U(z)$ is not a realizable function of $Y(z)$.

Outline

Disturbance Observer

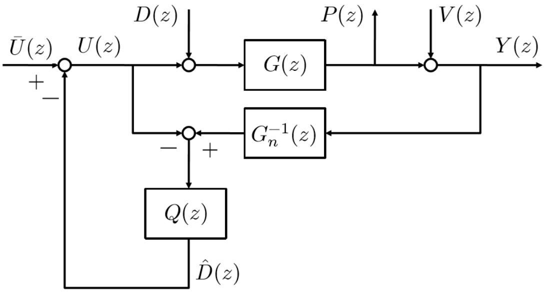
The following control structure is referred to as a disturbance observer:



The signals are:

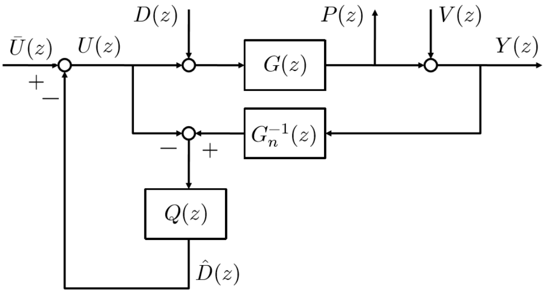
$$
\begin{aligned}
U(z) &: \text{control input} & V(z) &: \text{measurement noise} \\
D(z) &: \text{disturbance} & \hat{D}(z) &: \text{estimate of } D(z) \\
Y(z) &: \text{measured output} & P(z) &: \text{performance output}
\end{aligned}
$$

Disturbance Observer



- The one difference in the control architecture (compared to the motivation) is the presence of $Q(z)$
- $Q(z)$ is used to make the dynamics from $U(z)$ and $Y(z)$ to $\hat{D}(z)$ realizable

Disturbance Observer—Comparison to Motivation



The structure in the Motivation section corresponds to

- $G(z) = G_n(z)$ (the plant is exactly as modeled)

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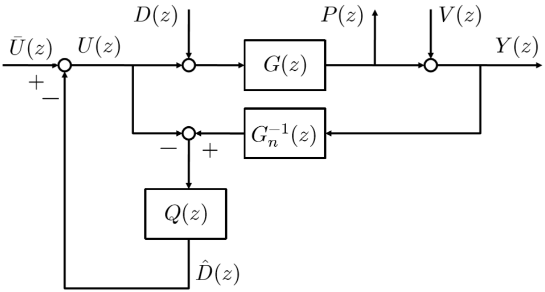

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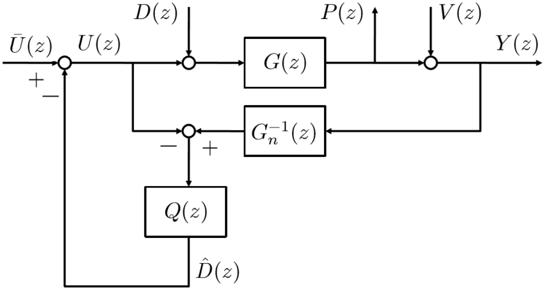

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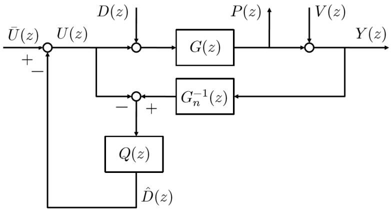
- $G(z) = G_n(z)$ (the plant is exactly as modeled)
- $V(z) = 0$ (there is no sensor noise)
- $Q(z) = 1$ (it is possible to realize $G_n^{-1}(z)$)

Outline

Derivation of closed-loop dynamics



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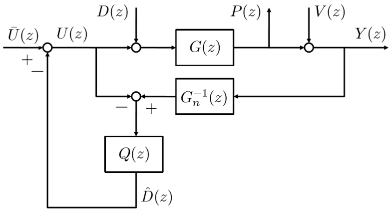

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Plant dynamics: $Y = G(U + D) + V$

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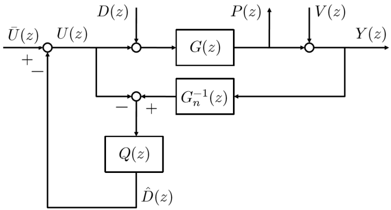

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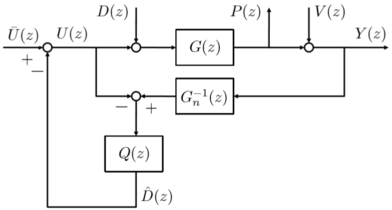

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Plant dynamics: $Y = G(U + D) + V$

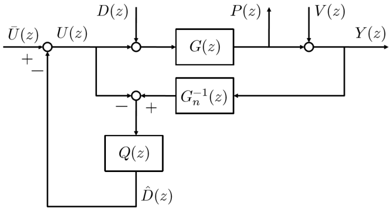
Now find the disturbance estimate $\hat{D}$ in terms of U, D, and V:

$$\hat{D} = Q(G_n^{-1}Y - U)$$

$$\Rightarrow \quad \hat{D} = Q[G_n^{-1}G(U + D) + G_n^{-1}V - U]$$

$$\Rightarrow \quad \hat{D} = Q(G_n^{-1}G - 1)U + QG_n^{-1}GD + QG_n^{-1}V$$

Derivation of closed-loop dynamics



Solve for U in terms of D, $\bar{U}$, and V:

$$U = \bar{U} - \hat{D}$$

$$\Rightarrow \quad U = \bar{U} - Q(G_n^{-1}G - 1)U - QG_n^{-1}GD - QG_n^{-1}V$$

$$\Rightarrow \quad [1 + Q(G_n^{-1}G - 1)]U = \bar{U} - QG_n^{-1}GD - QG_n^{-1}V$$

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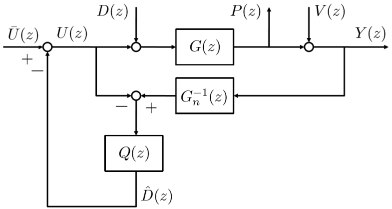

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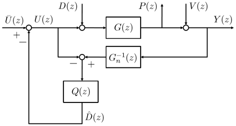
Now that we have U in terms of D, $\bar{U}$, and V, we can solve for P in terms of D, $\bar{U}$, and V

Derivation of closed-loop dynamics



Solve for P in terms of D, $\bar{U}$, and V:

$$P = GD + GU$$

$$\Rightarrow \quad P = GD + \frac{G}{1 + Q(G_n^{-1}G - 1)}[\bar{U} - QG_n^{-1}GD - QG_n^{-1}V]$$

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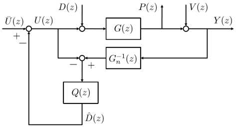

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$$\boxed{P = \frac{G(1-Q)}{1 + Q(G_n^{-1}G - 1)}D + \frac{G}{1 + Q(G_n^{-1}G - 1)}\bar{U} - \frac{GQG_n^{-1}}{1 + Q(G_n^{-1}G - 1)}V}$$

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$$

In forming this relationship, we used that $G_n G_n^{-1} = 1$, which in turn demonstrates why we require G_n to be minimum phase

Outline

Choosing $Q(z)$

Closed-loop dynamics:

$$\boxed{P = \frac{G_n(1+\Delta)(1-Q)}{1+Q\Delta}D + \frac{G_n(1+\Delta)}{1+Q\Delta}\bar{U} - \frac{Q(1+\Delta)}{1+Q\Delta}V}$$

Concerns when choosing $Q(z)$:

1. **Robust disturbance rejection:** Choose $Q(e^{j\omega}) \approx 1$ at frequencies for which disturbance rejection is important

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3. **Robustness:** Choose $|Q(e^{j\omega})|$ to be small at frequencies for which $|\Delta(e^{j\omega})|$ is large

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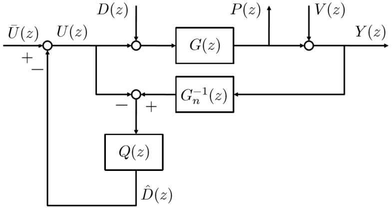
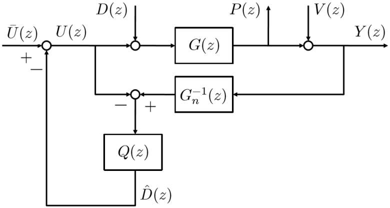


Concerns when choosing $Q(z)$:

4. **Realizability:** Choose $Q(z)$ so that $\hat{D}(z) = Q(z)[G_n^{-1}(z)Y(z) - U(z)]$ is realizable

Choosing $Q(z)$



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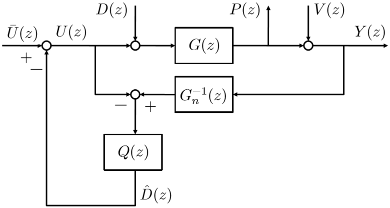

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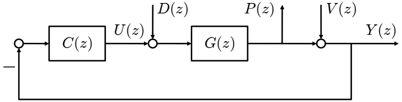
   This is a constraint on the relative degree of $Q(z)$

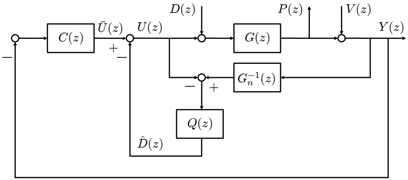
Outline

Adding a disturbance observer to an existing controller

Suppose we have designed a controller $C(z)$ for the interconnection



and we would like to add a disturbance observer:



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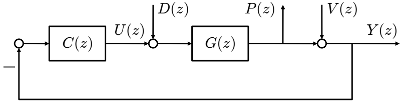

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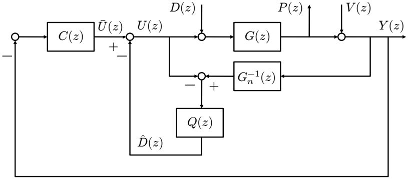
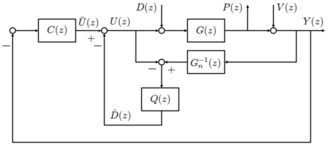


How does this affect the stability of the closed-loop system?

Adding a disturbance observer to an existing controller



Since we are only interested in stability, we set the exogenous inputs to zero. Also, we let $G(z) = G_n(z)(1 + \Delta(z))$.

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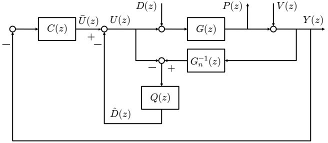

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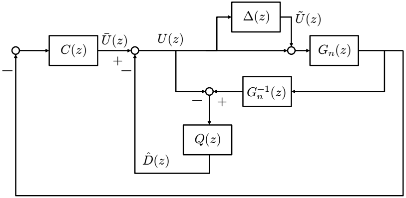

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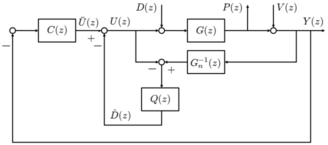

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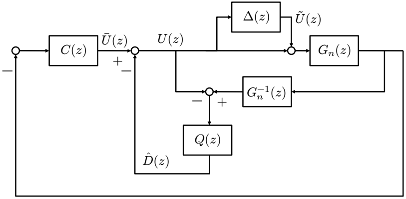
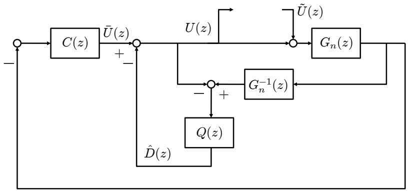


To use the small-gain theorem, we must simplify this to a feedback interconnection of $\Delta(z)$ and another system.

Simplifying the closed-loop representation

Removing $\Delta(z)$ from the interconnection, we have



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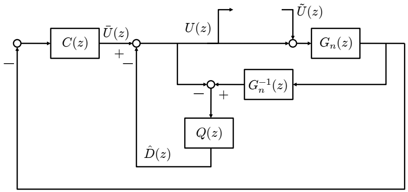


Omitting dependency on z, we have

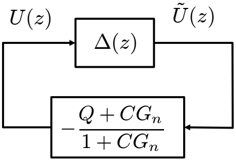
$$\hat{D} = Q\left[\frac{G_n}{G_n}(\tilde{U}+U) - U\right] \quad \Rightarrow \quad \hat{D} = Q\tilde{U}$$

$$U = -CG_n(\tilde{U}+U) - \hat{D} \quad \Rightarrow \quad U = -CG_n(\tilde{U}+U) - Q\tilde{U}$$

$$\Rightarrow \quad (1 + CG_n)U = -(CG_n + Q)\tilde{U}$$

Closed-loop stability

We now have the simplified closed-loop system representation



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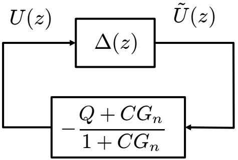

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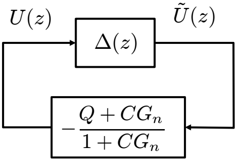

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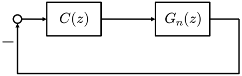
2. The following feedback interconnection is stable



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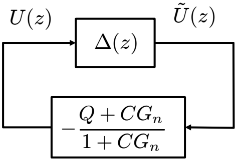

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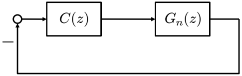

(i.e. the nominal closed-loop system without the disturbance observer is stable)

Closed-loop stability

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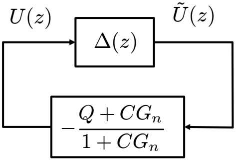

Using the small-gain theorem, we can therefore guarantee closed-loop stability if:

3. $$\left| \frac{Q(e^{j\omega}) + C(e^{j\omega})G_n(e^{j\omega})}{1 + C(e^{j\omega})G_n(e^{j\omega})} \right| < \frac{1}{|\Delta(e^{j\omega})|}, \quad \forall \omega \in [0, \pi]$$

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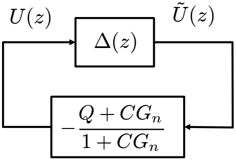

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   In order to meet this condition, it must be true that $Q(e^{j\omega}) \not\approx 1$ whenever $\omega \in [0, \pi]$ is such that $|\Delta(e^{j\omega})| \geq 1$.