UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME233 Advanced Control Systems II
Spring 2016

Homework #5

Assigned:	Apr. 6	(W)
Due:	Apr. 12	(Tu)

1. Figure 1 shows the feedback interconnection for a system with a disturbance observer. When implementing the disturbance observer, we only implement the portion that generates $U(z)$ from $\bar{U}(z)$ and $Y(z)$, as shown in Fig. 2.

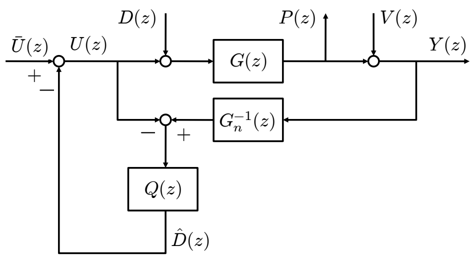

Figure 1: Disturbance Observer Structure

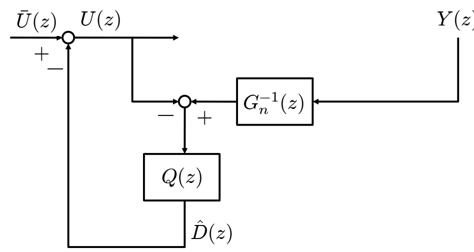

Figure 2: Disturbance Observer—Controller Only

(a) Find the transfer function from $Y(z)$ and $\bar{U}(z)$ to $U(z)$ in Fig. 2.

(b) Suppose that G_n^{-1} is proper. In this case, it is valid to choose $Q(z) = \alpha \in \mathcal{R}$. Based on your answer from the previous part, note that the block diagram in Fig. 2 is not well-posed when $\alpha = 1$. Does there exist $\alpha \in \mathcal{R}$ such that the closed-loop transfer function from $D(z)$ to $P(z)$ is zero? If not, is there a limit to how small we can make the transfer function from $D(z)$ to $P(z)$?

2. Consider the feedback system in Fig. 3 where $u(k)$ and $d(k)$ are respectively the

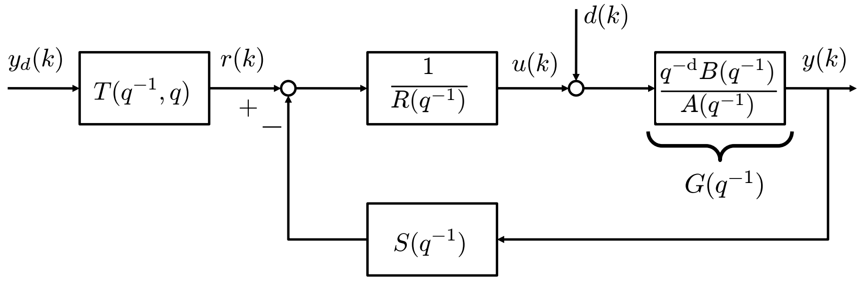

Figure 3: Feedback System

control and disturbance plant inputs, $y_d(k)$ is the reference model's output, and $r(k)$ is the reference input to the feedback block.

The control objective is to reject the persistent deterministic disturbance $d(k)$, place the feedback closed-loop poles, and track the desired output $y_d(k)$.

(a) The plant transfer function $G(z)$ is derived from a continuous time transfer function $G(s)$ that is preceded by a zero-order hold and followed by a sampler, and is given by

$$G(z) = \frac{\bar{B}(z)}{\bar{A}(z)} = (1 - z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right\},$$

where

$$G(s) = \frac{1}{s(s+1)}$$

and the sampling time is $T = 0.5$ seconds.

Calculate the plant polynomials $A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2}$, $B(q^{-1}) = b_o + b_1 q^{-1}$ and pure delay time d.

(b) The tracking control objective is to follow the reference signal $y_d(k)$, which is the output of the reference model

$$A_m(q^{-1}) y_d(k) = q^{-\mathrm{d}} B_m(q^{-1}) u_d(k)\,. \tag{1}$$

Select the coefficients of the second order polynomial $A_m(q^{-1}) = 1 + a_{m1}\, q^{-1} + a_{m2}\, q^{-2}$, so that the reference model has a natural frequency of 1 rad/sec and a damping ratio of 0.707.

Hint: Remember that, since $z = e^{sT}$, we can calculate the discrete time poles by $p_d = e^{p_c T}$, where p_d is the discrete time pole, p_c is the continuous time pole and T is the sampling time.

(c) Letting $B_m(q^{-1}) = b_{mo}$, select b_{mo} so that the reference model has unity static gain [1].

[1] i.e. if $\lim_{k\to\infty} u_d(k) = u_{ss}$ then $\lim_{k\to\infty} y_d(k) = u_{ss}$.

(d) Choose the coefficients of the closed-loop system characteristic polynomial (after pole-zero cancelation)

$$A_c^{'}(q^{-1}) = 1 + a_{c1}^{'}\, q^{-1} + a_{c2}^{'}\, q^{-2}$$

so that the closed-loop feedback dynamics from $r(k)$ to $y(k)$ behaves as a second-order system with a natural frequency of 2 rad/sec and a damping ratio of 0.5.

(e) Design the control system under the following specifications and assumptions:

i. The closed-loop system characteristic polynomial (before pole-zero cancelation) is given by

$$A_c(q^{-1}) = A_c^{'}(q^{-1}) B^s(q^{-1})$$

where

$$B^s(q^{-1}) = \frac{1}{b_o} B(q^{-1}), \qquad B^u(q^{-1}) = b_o\,,$$

and b_o is the leading coefficient of $B(q^{-1})$. This means that all of the plant zeros will be canceled by the feedback system.

ii. Assume that $d(k) = 0$. This means that the disturbance annihilating polynomial is selected to be $A_d(q^{-1}) = 1$.

iii. The feedforward compensator $T(q^{-1}, q)$ must be selected so that perfect tracking is achieved under a zero initial state for both the plant and the reference model.

(f) Do a computer simulation of the control system designed in problem 2e when $y_d(-1) = y_d(0) = y(-1) = y(0) = 0$ and

$$u_d(k) = [u_s(k) - 2\, u_s(k-25)] + [2\, u_s(k-50) - 2\, u_s(k-75)] \tag{2}$$

$$d(k) = 0.5 u_s(k-40) \tag{3}$$

where $u_s(j)$ is the unit step function, i.e.

$$u_s(j) = \begin{cases} 0 & j < 0 \\ 1 & j \geq 0 \end{cases}$$

Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(g) Design the control system under the same specifications in problem 2e, except that assume now that $d(k) = d(k-1)$.

(h) Do a computer simulation of the control system designed in problem 2g under the conditions described in problem 2f. Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(i) Design the control system under the following specifications and assumptions:

i. The closed loop system characteristic polynomial satisfies

$$A_c(q^{-1}) = A_c^{'}(q^{-1}) B^s(q^{-1})$$

where

$$B^s(q^{-1}) = 1, \qquad\qquad B^u(q^{-1}) = B(q^{-1}),$$

This means that none of the plant zeros will be canceled by the feedback system.

ii. Assume that $d(k) = d(k-1)$.

iii. The feedforward compensator $T(q^{-1}, q)$ is designed using the zero-phase error tracking control approach.

(j) Do a computer simulation of the control system designed in problem 2i under the conditions described in problem 2f. Plot $u_d(k)$, $y_d(k)$, $y(k)$ and $u(k)$.

(k) Discuss the outcome of the simulation results. In particular

- Comment on the effectiveness of the zero-phase feedforward control technique.
- Compare the control effort $u(k)$ when the zeros are canceled vs when the zeros are not canceled.